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On bounds for the complexity of hyperbolic 3-manifolds

Abstract. If M is a compact 3-manifold, its *complexity* [1] is a non-negative integer $c(M)$ which formally translates the intuitive notion of "how complicated" M is. In particular, if M is closed and irreducible and different from the 3-sphere, the projective 3-space, and the lens space $L(3,1)$, its complexity $c(M)$ is equal to the minimum of the number of tetrahedra over all "singular" triangulations of M.

The task of computing the complexity $c(M)$ of a given manifold M is very difficult. For closed M, the exact value is presently known only if M belongs to the computer-generated tables of manifolds up to complexity 12, see [1]. Therefore the problem of finding "reasonably good" two-sided bounds for $c(M)$ is of primary importance.

In the talk we will discuss the method used in [2] and [3] to establish two-sided bounds for the complexity of few infinite series of closed orientable hyperbolic 3-manifolds, the Löbell manifolds, the Fibonacci manifolds, and the cyclic branched coverings of 3-sphere, branched over 2-bridge links.

The upper bounds are obtained by constructing fundamental polytopes of these manifolds in hyperbolic space $\mathbb{H}^3$, while the lower bounds (which are proved in an "asymptotic" fashion) are based on the calculation of their volumes. We mention that the Löbell manifolds are constructed from polytopes that generalize the right-angled dodecahedron, and the Fibonacci manifolds are constructed from polytopes that generalize the regular icosahedron.

References

[1] S. V. Matveev, *Algorithmic topology and classification of* 3*-manifolds*, ACM-monographs Vol. 9, Springer-Verlag, Berlin-Heidelberg-New York, 2003.
[2] S. Matveev, C. Petronio, A. Vesnin, *Two-sided asymptotic bounds for the complexity of some closed hyperbolic three-manifolds*, J. Australian Math. Soc. 86(2) (2009), 205-219.
[3] C. Petronio, A. Vesnin, *Two-sided asymptotic bounds for the complexity of cyclic branched coverings of two-bridge links,*. Osaka J. Math. 46(4) (2009).