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On volumes and normalized volumes of right-angled hyperbolic polyhedra

Abstract: Let $\mathcal{R}$ be the class of right-angled polyhedra in hyperbolic space $\mathbb{H}^3$. Denote by $\mathrm{vert}(R)$ the number of vertices of a polyhedron R and by $\mathrm{vol}(R)$ its volume. Explicit formulae for volumes of right-angled hyperbolic polyhedra are known for few families only (see [1]).

In last two years some interesting results on volumes of right-angled hyperbolic polyhedra were obtained. Inoue [2] introduced two operations, *decomposition* and *edge surgery*, on compact polyhedra from $\mathcal{R}$ which admit to reduce any polyhedron to a set of Löbell polyhedra introduced in [1]. Two-sided estimates for volume of polyhedra from $\mathcal{R}$ in terms of number of vertices were obtained in [3]. By *normalized volume* of a hyperbolic polyhedron R we will mean the value $\omega(R) = \mathrm{vol}(R)/\,\mathrm{vert}(R)$.

We will discuss the behavior of $\omega(R)$ for various classes of polyhedra in $\mathbb{H}^3$. In particular, under the operations defined in [2]. We will show that upper estimates from [3] for volumes of hyperbolic polyhedra are related to limits of $\omega(R)$ on suitable families of polyhedra [4]. Also, low bounds form [3] will be improved.

References

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