**Martin TANCER**

## Hardness of embedding simplicial complexes in $\mathbb{R}^d$

*Abstract.* Let $\mathrm{EMBED}_{k\to d}$ be the following algorithmic problem: Given a finite simplicial complex $K$ of dimension at most $k$, does there exist a (piecewise linear) embedding of $K$ into $\mathbb{R}^d$? Known results easily imply polynomiality of $\mathrm{EMBED}_{k\to 2}$ ($k = 1, 2$; the case $k = 1$, $d = 2$ is graph planarity) and of $\mathrm{EMBED}_{k\to 2k}$ for all $k \geq 3$ (even if $k$ is not considered fixed).

The main result presented in the talk is NP-hardness of $\mathrm{EMBED}_{2\to 4}$ and, more generally, of $\mathrm{EMBED}_{k\to d}$ for all $k, d$ with $d \geq 4$ and $d \geq k \geq (2d-2)/3$. We also show that the celebrated result of Novikov on the algorithmic unsolvability of recognizing the 5-sphere implies that $\mathrm{EMBED}_{d\to d}$ and $\mathrm{EMBED}_{(d-1)\to d}$ are undecidable for each $d \geq 5$.
(This is joint work with Jiří Matoušek and Uli Wagner.)