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Extension of pseudometrics in a zero dimensional space

Abstract. The problem of extension of a metric was initially considered by Felix Hausdorff in the early 30-s of the previous century. He was the first to show that every metric compatible with the topology of a closed subspace of a metrizable topological space can be extended to a compatible metric on the whole space. Various generalizations of Hausdorff's theorem have been proved by many authors since that time.

The problem of existence of linear operators extending the cone of (pseudo)metrics was stated and solved for some special cases by C. Bessaga. T. O. Banach obtained the complete solution of the mentioned problem. Recently E. D. Tymchatyn and M. M. Zarichnyi constructed continuous (in the Vietoris topology) operators which simultaneously extend continuous pseudometrics (ultrametrics) defined on closed subsets of a metrizable compact (zero-dimensional) topological space. The extension operator for pseudometrics has the additional property of linearity and that for ultrametrics preserves the maximum of two ultrametrics with a common domain and the Assouad dimension of the ultrametric space. Both constructions essentially depend on the existence of continuous selections of certain multivalued maps. A modification of Tymchatyn-Zarichnyi construction allows us to obtain a homogeneous operator extending partial ultrametrics which has all the original properties.

We consider the problem of simultaneous extension of continuous pseudometrics defined on closed subsets of a metrizable zero-dimensional locally compact space. The set of partial pseudometrics is endowed with the Fell topology. We show that an extension operator constructed by applying Michael's selection theorem for zero-dimensional spaces is continuous on the sets of uniformly bounded and uniformly equicontinuous on compact subsets of partial pseudometrics.