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Bravais-lattices and their visualization in Sol space

Abstract. Sol geometry is one of the eight homogeneous Thurston 3-geometries

$$\mathbf{E^3}, \mathbf{S^3}, \mathbf{H^3}, \mathbf{S^2} \times \mathbf{R}, \mathbf{H^2} \times \mathbf{R}, \widetilde{\mathbf{SL_2R}}, \mathbf{Nil}, \mathbf{Sol}.$$

In our talk we classify **Sol** lattices in an algorithmic way into 17 (seventeen) types, infinitely many equivalence classes in each, in analogy of the 14 Bravais types of Euclidean 3-lattices. To this we study relations between **Sol** lattices and lattices of the pseudoeuclidean (or Minkowskian) plane [1].

Moreover, we determine the densest translation ball packing [4] by so-called fundamental lattices which is one (Type **I/1**) of the 17 Bravais-type of **Sol**-lattices described in [2]. The optimal arrangement will have richer symmetry group (in Type **II/2**) for N = 4. This density is $\delta \approx 0.56405083$ and the kissing number of the balls to this packing is 6. We intend to find the locally densest ball packings to every lattice type.

We use the affine model of **Sol** space through affine-projective homogeneous coordinates which gives a unified way of investigating and visualizing homogeneous spaces, in general.

References

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[3] Scott, P. *The geometries of 3-manifolds.* Bull. London Math. Soc., 15 401487, (1983). (Russian translation: Moscow Mir 1986.)

[4] Szirmai, J. *The densest translation ball packing by fundamental lattices in* **Sol** *space.* Manuscript to Beiträge zur Algebra und Geometrie (Contributions to Algebra and Geometry), 2009.