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Special tetrahedron manifolds derived from combinatorial cube

Abstract: I intend to sketch a topic of joint investigations of Hungary and Slovenia in the framework of our joint project. I shall start with an earlier computational result of Istvan Prok who listed (among others) the 37 fixed point free face pairings of a combinatorial cube preserving the orientation (up to cube automorphisms). The general polyhedron algorithm was described and analyzed by myself, in the analogy with Poincare. I have now collected such 9 cube pairings, where at least one induced edge equivalence class consists of 2 edges, resulting equal generators, so that our combinatorial cube degenerates to a "tetrahedron" with artificial vertices. The induced equivalence for vertices can be pictured by surface diagram. Thus we obtain cases where the vertex stabilizer will be trivial (the surface diagram will be the 2-sphere or the vertex stabilizer is the usual E2-lattice group with a torus as surface diagram. Then the two translation generators can be expressed by the original face pairing generators and the linear algebraic presentation by the machinery of the projective 3-sphere can provide hyperbolic metric realizations with additional "surgery" relations for resulting compact hyperbolic (in most cases) manifolds or cusped manifolds. There are easier (didactically important) cases where the edge class relations yield finite groups since only S3-realizations are possible, although the concrete computations by the machinery are not trivial. Then are extra cases where splitting effect occurs, along a two-dimensional Klein bottle or along a projective plane, where the resulting pieces may wear different Thurston metrics, etc.

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