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Elementary surgery on 4-manifolds

Abstract. The well known Dehn surgery on a 3-dimensional manifold M embeds a solid torus $D^2 \times S^1$ into a 3-manifold M, making a thickend knot, then removes the interior, and pastes it back via a diffeomorphism $f : \partial D^2 \times S^1 \to \partial D^2 \times S^1$. Dehn surgery is described by the knot (the core of the embedded solid torus) and a rational number (or ∞) called the 'slope'. There are several 4-dimensional analogies, for example, Gluck surgery, which uses $S^2 \times D^2$ in place of a solid torus. This surgery is described by the 2-knot (the core of the embedded $S^2 \times D^2$) and a $\mathbb{Z}_2$-framing. We develop another type of a 4-dimensional surgery using a 'pochette '$P = S^1 \times D^3 \natural D^2 \times S^2$ (boundary connected sum of $S^1 \times D^3$ and $D^2 \times S^2$). Although the boundary of a pochette ∂P is diffeomorphic to $S^1 \times S^2 \# S^1 \times S^2$, and has a non-commutative fundamental group, the result of our pochette surgery is descibed by the slope belonging to $\mathbb{Q} \cup \{\infty\}$ and a $\mathbb{Z}_2$-framing. Also in our surgery, there is a geometric operation closely related to the continued fractional expression of the slope, just as in the classical Dehn surgery. An application to branched coverings of S^4 will be exlpained. This is a joint work with Zjuñici Iwase at Kanazawa University.