Nadya LYASKOVSKA

The packing index of measurable subsets in locally compact groups

Abstract: The famous problem of optimal sphere packing traces its history back to B.Pascal and belongs to the most difficult problems of combinatorial geometry. In this talk we consider an analogous problem in the algebraic setting. Namely, given a non-empty subset A of an Abelian group G we study the cardinal number

$$\mathrm{ind}_P(A) = \sup\big\{|B| : B \subset G \text{ and } (B-B)\cap(A-A) = \{0\}\big\}$$

called the *packing index* of A in G.

Theorem. *An infinite Abelian group G contains a subset $A \subset G$ with packing index $ind_P(A) = \kappa$ if and only if one of the following conditions holds:*

1) $1 \le \kappa \le |G|$ *and* $\kappa \notin \{2,3\}$.
2) $\kappa = 2$ *and G is not isomorphic to* $\oplus_{i\in I}\mathbb{Z}_3$.
3) $\kappa = 3$ *and G is not isomorphic to* $\oplus_{i\in I}\mathbb{Z}_2$ *or to* $\mathbb{Z}_4 \oplus (\oplus_{i\in I}\mathbb{Z}_2)$.

Moreover if G is locally compact and not discrete then the set $A \subset G$ with $ind_P(A) = \kappa$ can be chosen to be nowhere dense and of zero Haar measure in G.

Thus measurable subset of locally compact Abelian groups can have the packing index intermediate between $\aleph_0$ and $\mathfrak{c}$. This is not the case for σ-compact subsets

Theorem. *Let G be Polish group and $A \subset G$ be σ-compact then $ind_P(A) \in \omega \cup \{\omega, \mathfrak{c}\}$.*