Friedrich HEGENBARTH

Homological aspects of Surgery Theory

Abstract: Surgery Theory goes back to the late 50th. Surgeries on manifolds or spherical modifications, as they were called at the beginning, are used to modify a given manifold M by attaching handles to $M \times I$, called trace of the surgery. Its boundary has two components, M and M'(i.e. M' is obtained from M by surgeries or spherical modifications). Morse Theory says that two manifolds are cobordant if they are related by spherical modifications.

The breakthrough of Surgery Theory came with the paper of Kervaire and Milnor (1961) on exotic homotopy spheres. Parallel to it, Browder and Novikov extended the Kervaire-Milnor results to simply-connected manifolds. In the middle of the 60s, C.T.C.Wall introduced general surgery obstruction groups (depending only on the fundamental group) and established a 4-term exact sequence, later called the surgery sequence. This sequence enables one to calculate (exotic) structures on manifolds and Poincaré complexes.

D.Sullivan and F. Quinn refined Surgery Theory by breaking a given surgery problem into small pieces. This led finally to the construction of a surgery spectrum, called L-spectrum. The surgery obstruction groups of Wall can be replaced by L-homology groups of a space. E.Pedersen, F.Quinn and A.Ranicki, as well as S.Ferry (around 2003/4) established an exact surgery sequence having L-homology groups of a space B as surgery obstruction groups provided there is a UV^1 map of M onto B. Building on their fundamental work, we shall show that the Pedersen-Quinn-Ranicki sequence is the L-homology sequence of the pair (B, M).