ALGEBRAIC TOPOLOGY OF ONE DIMENSIONAL CONTINUA

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Well-known and well-used one dimensional continua are finite graphs. Finite graphs are contractible or homotopy equivalent to finite bouquets and so their fundamental groups are isomorphic to free groups of finite rank and their homology groups are free abelian groups of finite rank and hence algebraic topology of them are easy to understand. On the other hand, complicated one dimensional continua were also known for a long time. But, complicated one dimensional continua have attracted attention with the words "Chaos" and "Fractal". They appear as limit objects in several scenes in mathematics and also in physics.

To state the contents of Lectures, I explain some terminology. Continua, here I mean, are connected compact metric spaces. Peano continua are locally path-connected continua. Complexities occur when some local properties are lacked, i.e. local path-connectivity and semi-local simple-connectivity.

Fundamental groups of Peano continua determine homotopy types of them and its proofs heavily depending on the non-commutative Specker phenomenon. In the abelian group theory there are dual notions, in a loose sense. That is, slender groups are related to the Specker phenomenon and algebraically compact groups are at the opposite side.

Lecture 1: The Čech homotopy (shape) groups of one dimensional continua.

For Peano continua, the shape types of them are classified by finite bouquets and the Hawaiian earring and consequently Čech homotopy groups are classified by free groups of finite rank and that of the Hawaiian earring. Here I outline proofs of the classification of Čech homotopy groups of one dimensional continua. Actually there exist three other groups for them and the classification is the same for separable metric connected one dimensional spaces. The proofs use the Specker phenomenon for non-commutative groups and abelian groups and also the algebraical compactness [5].

(1) Group theoretic statements;
(2) Realizations of the three groups;
(3) Non-isomorphicness of the groups in list.

Lecture 2: The singular homology groups of one dimensional Peano continua.

It is well-known the singular homology groups H_n for $n \geq 2$ of one dimensional spaces vanish. Hence H_1 is the only question and is the abelianization of the fundamental group of a path-connected space. It turns out that the one dimensional singular homology groups of one dimensional Peano continua are classified to free abelian groups of finite rank and that of the Hawaiian earring. A proof heavily depends on the algebraical compactness in abelian group theory, which is in a counter part of the Specker phenomenon. [3]

(1) Complete mod-U groups and algebraically compact groups;
(2) The canonical homomorphism from the singular homology to the Čech one;
(3) Local construction and global one.

Lecture 3: Infinite sheeted connected covering spaces over solenoids.

Solenoids are one dimensional connected compact abelian groups distinct from the circle group $\mathbb{R}/\mathbb{Z}$. Finite sheeted covering maps over a solenoid having connected total spaces are always equivalent to homomorphisms, which are the Pontrjagin dual of inclusion maps between subgroups of the rational groups $\mathbb{Q}$. More precisely, we can induce a compatible group structure to the total space making the map as a homomorphism. Hence, through the Pontrjagin duality the total spaces of finite sheeted covering homomorphisms corresponds to the finite index super groups of a rational group. A natural question was what happens in the infinite sheeted case. In our jointed paper, Vlasta Matijevic has proved that this is not the case. That is, an infinite sheeted covering map over a solenoid never become a homomorphism. In addition she constructed an infinite sheeted covering map over a dyadic solenoid with a very complicated construction. In this lecture a simple construction using an elementary number theory is presented [4].

(1) Covering homomorphisms and overlays;
(2) Presentation of solenoids;
(3) Construction of infinite sheeted covering maps.

References

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