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Covering maps for locally path-connected spaces

Abstract. We define Peano covering maps and prove basic properties analogous to classical covers. Their domain is always locally path-connected but the range may be an arbitrary topological space. One of characterizations of Peano covering maps is via the uniqueness of homotopy lifting property for all locally path-connected spaces.

Regular Peano covering maps over path-connected spaces are shown to be identical with generalized regular covering maps introduced by Fischer and Zastrow. If X is path-connected, then every Peano covering map is equivalent to the projection $\tilde{X}/H \rightarrow X$, where H is a subgroup of the fundamental group of X and \tilde{X} equipped with the basic topology. The projection $\tilde{X}/H \rightarrow X$ is a Peano covering map if and only if it has the unique path lifting property. We define a new topology on \tilde{X} for which one has a characterization of $\tilde{X}/H \rightarrow X$ having the unique path lifting property if H is a normal subgroup of $\pi_1(X)$. Namely, H must be closed in $\pi_1(X)$. Such groups include $\pi(\mathcal{U}, x_0)$ (\mathcal{U} being an open cover of X) and the kernel of the natural homomorphism from the fundamental group to the Čech fundamental group.