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A pointwise selection principle for functions of a real variable and selections of bounded variation

Abstract. The classical Helly selection principle asserts that a uniformly bounded sequence of monotone real functions on a closed interval contains a pointwise convergent subsequence. There is a number of generalizations of this principle for real (or even metric space valued) functions of bounded (generalized) variation in the sense of Jordan, Wiener and Young, Waterman, Gnilka, Schramm, etc. We present a unifying approach to the pointwise selection principle for sequences of functions mapping a subset of the real line into a metric space (or even a Hausdorff uniform space). The main condition in this approach is expressed in terms of a certain growth of the modulus of variation in the sense of Chanturiya of the functions from the sequence, which we study in detail. We show that our approach gives all the above mentioned Helly type selection principles. As an application of our pointwise selection principle, we show that a multifunction of bounded Jordan variation, whose domain is a subset of the real line and the values are nonempty compact (not necessarily convex) subsets of a metric space, admits a selection of bounded variation, passing through a given point in the graph of the multifunction and whose total Jordan variation does not exceed the total Jordan variation of the multifunction calculated in the respective Hausdorff metric.