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Constructing 3-manifolds

Abstract: The twisted face-pairing construction gives an efficient way of generating, mechanically and with little effort, myriads of relatively simple face-pairing descriptions of interesting closed 3-manifolds. The corresponding description in terms of surgery, or Dehn-filling, reveals the twist construction as a carefully organized surgery on a link. In this talk, we work out the relationship between the twisted face-pairing description of closed 3-manifolds and the more common descriptions by surgery and Heegaard diagrams. We show that all Heegaard diagrams have a natural decomposition into subdiagrams called Heegaard cylinders, each of which has a natural shape given by the ratio of two positive integers. We characterize the Heegaard diagrams arising naturally from a twisted face-pairing description as those whose Heegaard cylinders all have integral shape. This characterization allows us to use the Kirby calculus and standard tools of Heegaard theory to attack the problem of finding which closed, orientable 3-manifolds have a twisted face-pairing description. (Joint work with W.J. Floyd and W.R. Parry)