Maxim BALASHOV

Complementary of the set to the body of fixed width

Abstract: We consider next old problem: for given bounded subset A from Banach space E find the body of fixed width $= \operatorname{diam} A$ which contains A. We will formulate the concept of generating set. On the base of this concept we will discuss the results of E. Polovinkin et al.

In Banach space E we define $B_d(a) = \{x \in E \mid \|x - a\| \le d\}$.

T h e o r e m 1. *(E. Polovinkin) Let E be a Banach space with generating ball (for example Hilbert space). Let $A \subset E$ and* $\operatorname{diam} A = d > 0$. *Let*

$$M_d(A) = \bigcap_{a \in A} B_d(a), \qquad m_d(A) = M_d(M_d(A)) = \bigcap_{b \in \bigcap\limits_{a \in A} B_d(a)} B_d(b).$$

Then the set $W(A) = \frac{1}{2}(M_d(A) + m_d(A))$ is the body of fixed width $= d$, which contains A.

C o r o l l a r y 1. *(E. Polovinkin, D. Sidenko) Let E be a Banach space with generating ball and $A \subset E$,* $\operatorname{diam} A = d > 0$. *Then every set W of fixed width $= d$, $W \supset A$, satisfies the inclusions*

$$m_d(A) \subset W \subset M_d(A).$$

C o r o l l a r y 2. *(M. Balashov) If E is Hilbert space and $A \subset E$ is smooth convex set (i.e. at any boundary point of the set A there exists the only one supporting plane), then the set $W(A)$ is smooth too.*