Lifting knots and links from lens spaces to the 3-sphere

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Abstract

A new diagram for knots and links in lens spaces has recently been introduced in [CMM]. Namely, consider the lens space $L(p,q)$ as the quotient of the unit ball B^3 where each boundary point is identified with the one in the opposite hemisphere after a planar reflection and a rotation of $\frac{2\pi q}{p}$ radians around the polar axis. We can project any link on the equatorial disk of B^3, obtaining a regular diagram for it.

The main topic of the talk is the lift of links in lens spaces: given a link L in $L(p,q)$, and assigned the cyclic covering map $P : \mathbf{S}^3 \to L(p,q)$, the lift $\widetilde{L}$ of L is the counterimage $P^{-1}(L) \subset \mathbf{S}^3$. In order to study properly its features, we have developed a geometric algorithm that, starting from a diagram of the link L in the lens space, allows us to recover a diagram of the corresponding lift $\widetilde{L}$ in $\mathbf{S}^3$.

With this construction we have been able to find different knots and links in $L(p,q)$ with the same lift, that is to say, we cannot distinguish inequivalent links in lens spaces only from their lift. As a consequence, we investigated if several invariants are or not stronger than the lift.

References

[CMM] A. Cattabriga, E. Manfredi and M. Mulazzani, *On knots and links in lens spaces*, Topology Appl. **160** (2013), 430442.