INTRODUCTION TO INFINITARY WORDS AND PATHS IN ONE DIMENSIONAL SPACES

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As a model of the fundamental group of the Hawaiian earring we can imagine an infinitary version of usual words of finite length. Here our emphasis is an infinitary version of edge-path groups. We regard paths in one dimensional spaces as infinitary version of edge-paths, i.e. every point in the space is a vertex. Here reduced paths correspond to reduced words in the edge-path group. A fundamental tool is the 0-form lemma, which show what is a form of the reduced loop whose homology class is trivial (we fill a gap in the proof of the corresponding lemma in [E1]). As applications of this point of view we demonstrate to show the following basic facts.

1. The first singular homology group of a path-connected subspace of a one-dimensional space X is a subgroup of that of X.

2. The first singular homology group of the Hawaiian earring contains a free abelian group of the continuum rank as a pure subgroup.

3. The first singular homology group of the Griffiths twin cone contains a free abelian group of the continuum rank as a pure subgroup.

Reference.

[E1] K. Eda, Free σ-products and noncommutatively slender groups, J. Algebra **148** (1992), 243–263.

[E2] K. Eda, The fundamental groups of one-dimensional spaces and spatial homomorphisms, Topology Appl. **123** (2002), 479–505.