

**INTRODUCTION TO NON-STANDARD ANALYSIS:
CONNECTIONS AMONG ASYMPTOTIC CONES,
TROPICAL GEOMETRY AND THE MASLOV
CORRESPONDENCE**

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Gromov's asymptotic cone was clearly defined by van den Dries and Wilkie [1] using ultraproducts and has been studied by many authors, but the emphasis is sometimes related to the fact that the construction depends on choosing ultrafilters. Consequently thinking of nonstandard analysis does not seem to be useful in this area. On the other hand in tropical algebraic geometry many logical correspondences have been proved directly and thinking of nonstandard analysis is also abandoned. Similar thinking appears in the ultra-discrete analysis of D. Takahashi [2] and the heuristic correspondence of V. Maslov [3]. Here we give a point of view how they are connected and show what have been thought as analogies, i.e. analogical correspondences, are actually (mathematically) logical ones.

We explain how the characteristic equations of constant co-efficient differential equations and those of difference equations are related. Before explaining these, as an direct introduction of applications of nonstandard analysis we give a proof of the existence of a finite-additive measure on a group from the Følner sequence related to amenability.

Reference.

[1] L. van den Dries and A. Wilkie, Gromov's theorem on groups of polynomial growth and elementary logic, *J. Algebra* **89** (1984), 349–374.

[2] G.L. Litvinov and V.P. Maslov, The correspondence principle for idempotent calculus and some computer applications, *Idempotency* (J. Gunawardena, ed.), Cambridge Univ. Press, Cambridge, 1998, pp. 420–443.

[3] R. Hirota and D. Takahashi, *Difference and Ultra-discreteness*, Kyoritsu-shuppan, 2003, in Japanese.

[4] O. Viro, From the sixteenth Hilbert problem to toropical geometry, *Jap. J. Math.* **3**, (2008) 185–214.