Leonard R. Rubin
University of Oklahoma, Norman, USA
lrubin@ou.edu

Universal Spaces in Cohomological Dimension Theory

Abstract. Let $\mathcal{C}$ be a class of compact metrizable spaces. An element $Z \in \mathcal{C}$ is called *universal* for $\mathcal{C}$ if each element of $\mathcal{C}$ embeds topologically in Z. It is a well-known, classical result of dimension theory that for each $n \in \mathbb{N}$, there exists a universal element for the class of metrizable compacta X of (covering) dimension $\dim X \leq n$. Modern techniques involving the Stone-Čech compactification and the Mardešić factorization theorem yield relatively easy, albeit abstract, proofs of this result.

There is a parallel theory of dimension called cohomological dimension; indeed there is one such theory for each abelian group G. Although these theories concur with dimension in many ways, they do not in general agree with it. One may ask about the existence of universal compacta for this type of dimension. But it turns out that not all the techniques that work for dim apply to cohomological dimension; in particular one cannot use the Stone-Čech compactification at what would be a critical point of such a proof. It has thus been speculated that in most cases there do not exist universal compacta in the theory of cohomological dimension. We are going to speak about techniques that should lead to a proof of this nonexistence conjecture.