Simplicialne resolvente

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Naj bo $\mathfrak{m} \trianglelefteq R = K[\mathbf{x}] = K[x_1, \ldots, x_n]$ monomski ideal. Tedaj je $\mathfrak{m}$ generiran s končno mnogo monomi $\mathbf{x}^{\mathbf{a}_1}, \ldots, \mathbf{x}^{\mathbf{a}_m}$, kjer so $\mathbf{a}_1, \ldots, \mathbf{a}_m \in \mathbb{N}^n$ med seboj paroma neprimerljivi vektorji (glede na običajno delno ureditev po komponentah $\mathbf{a} \leq \mathbf{b} \Leftrightarrow \forall i\colon a_i \leq b_i$). Potem ima R-modul $R/\mathfrak{m}$ *Taylorjevo resolvento*

$$R^{(\{e_{[m]}\})} \longrightarrow \ldots \longrightarrow R^{(\{e_\sigma;\, \sigma \in \binom{[m]}{k}\})} \longrightarrow R^{(\{e_\sigma;\, \sigma \in \binom{[m]}{k-1}\})} \longrightarrow \ldots \longrightarrow R^{(\{e_\emptyset\})} \xrightarrow{\pi} R/\mathfrak{m} \longrightarrow 0,$$
$$e_\sigma \longmapsto \textstyle\sum_{i \in \sigma} (-1)^{\sigma(i)} \frac{\mathrm{lcm}_\sigma}{\mathrm{lcm}_{\sigma \setminus \{i\}}} e_{\sigma \setminus \{i\}} \text{ kjer je } \sigma(i) \text{ položaj } i \text{ v } \sigma.$$

Vsak simplicialni kompleks Δ na $[m]$ določa verižni podkompleks v Taylorjevi resolventi, ki pa ni nujno eksakten. Ogledali si bomo nekaj pomembnih primerov takih podkompleksov (Lyubeznik, Scarf).

References

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