Steenrod L-homology and representation of elements

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Abstract

Steenrod generalized homology theories are extensions of generalized homology theories defined on (CW-)complexes, to the category of compact metric spaces and continuous maps. We will briefly describe their construction, following Kahn-Kaminker-Schochet [2] and Ferry [1].

Our main interest is in L-homology, where L denotes the surgery spectrum. A surgery problem with target a manifold is classified by an element of L-homology group. This is due to transversality: if X is a generalized manifold, X is a compact ENR, which implies that X is metrizable, hence X is a compact metric space, so transversality cannot be applied. However, in this case we will associate to X an element of a Steenrod L-homology group. In order to construct such an element we shall have to explain additional topics:

- nerves and the fundamental complex of X,

- locally finite homology theories, and

- relations between L and normal bordism spectra.

References

[1] Ferry, Steven C. Remarks on Steenrod homology. *Novikov conjectures, index theorems and rigidity, Vol. 2 (Oberwolfach, 1993)*, 148–166, London Math. Soc. Lecture Note Ser., 227, *Cambridge Univ. Press, Cambridge*, 1995.

[2] Kahn, Daniel S.; Kaminker, Jerome; Schochet, Claude. Generalized homology theories on compact metric spaces. *Michigan Math. J.* **24** (1977), no. 2, 203–224.